Collective bargaining in the public sector and the role of budget determination

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Abstract

This paper considers collective bargaining in a public sector institutional setting. The model demonstrates how budget constraints and hierarchical structure affect the bargaining outcome. A trade union bargains over wage and employment either with an output-maximizing bureau or the bureau’s sponsoring institution. The slope of the contract curve depends on the bargaining level because the budget constraints differ. Various assumptions are made about the timing of the sponsor’s decision concerning the budget of the bureau. Local bargaining and budget determination between the wage and employment bargains can be optimal for the sponsor because it yields a low wage. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

Public sector trade unions are often regarded as influential. However, despite the large body of theoretical literature on collective bargaining, few papers have considered a public sector institutional setting explicitly. This is necessary to discover and interpret differences in collective bargaining outcomes between the public and private sector. Empirical results from the US indicate lower union–nonunion wage differentials in the
public sector than in the private sector (see the surveys in Ehrenberg and Schwarz, 1986; Freeman, 1986). If public sector trade unions are as influential as their private sector counterparts, then they must extract more of other kinds of rents, such as higher employment. This paper argues that the hierarchical structure of the public sector can lead to high union employment even though the union has low bargaining power.

I take into account several aspects that distinguish public and private sector labor markets. First, the decision-making environments differ. Total available income limits the production in the public sector in contrast to product demand in the private sector. Dunlop (1958) clearly describes the analogy between the market and budgetary constraints. The specification of the budget constraint is important in all public sector models with optimization, including labor market models. Strøm (1999) discusses wage bargaining in local governments, distinguishing between local financing and cash limits from the central government. With cash limits, a sponsoring institution determines the wage costs. Holmlund (1997) investigates the macroeconomic effects of trade union influence in both the public and private sectors when the public sector bureaus are constrained by cash limits. Modeling public sector wage bargaining in a cash-limit framework was introduced by Leslie (1985). With labor as the only input, cash limits imply that the bargaining parties face an elasticity of labor demand equal to one in absolute value. There is no room for bargaining over employment. The model developed in the present paper includes non-labor expenditures in order to take possible bargaining over employment into account.

Babcock and Engberg (1997) include additional relevant cost elements in a model where both the employment level and the budget size are given. In their model, a union induced wage increase is financed either by reallocating money from non-labor expenditures toward wage costs, or by reducing hiring and turnover costs by hiring less productive workers. An underlying premise of the approach is that employment is less flexible than wages, a realistic characteristic for many public sector services. Changing or restructuring a public service usually requires a long preparation process, often involving political controversies. The common view for the private sector where the variation in employment is larger, however, is that employment is the more flexible variable.1

Public sector trade unions are typically important actors in public debates about the content and organization of public services. These issues are important for the employment level. It can therefore be a reasonable simplification to assume that the unions have bargaining power over employment. In addition, recent evidence from aggregate data supports this assumption (see Alogoskoufis and Manning, 1991; de la Croix et al., 1996). However, the union influence over employment differs from the influence over wages. In contrast to the employment determination, wage bargaining is formalized to occur with given time intervals. In the sequential bargaining framework introduced by Manning (1987a,b), bargaining over the wage and employment take place at different times and in different environments (see also Pencavel, 1991). Collective bargaining is a

1 Notice that the assumption of employment as more flexible than wages in the private sector is not a universal assumption (see, for example, Grout, 1984; Horn and Wolinsky, 1988; Moene, 1988).
two-stage process. I assume throughout the paper that the employment decision is made prior to the wage decision, and that the bargaining parties have different relative bargaining powers in the wage and employment bargains. One special case of the model is that the union has no bargaining power over employment. The widely used efficient bargaining model, simultaneous bargaining over the wage and employment, is another special case of the model. The efficient bargaining model is discussed by Inman (1981) in a public sector context, and for example, McDonald and Solow (1981) in a private sector context. I show that the tradeoff between the wage and employment along the contract curve is less favorable for public sector unions than for private sector unions.

Lastly, a hierarchical structure is more pronounced in the public sector than in the private sector, presumably because the public sector is a large employer. OECD (1994) describes the wage determination systems for the national public administrations and argues that countries with a single central bargain have better budgetary control than countries with more decentralized bargaining. One possible explanation is that if “collective bargaining is to be in good faith, then there is necessarily some unpredictability in the final budgetary consequences” (OECD, 1994, p. 17). Smaller budgetary control with local bargaining implies that the government sets the final budget after the wage determination. Compared to a situation without union influence, the local level can more easily come into a leader position in the budgetary game when unions have bargaining power. The result of union influence can therefore be high employment even though there is a low union–nonunion wage mark-up. For the local public sector in the US, Freeman (1986) and Zax and Ichniowski (1988) among others, report a positive relationship between the employment level and the occurrence of union bargaining. While these authors argue that labor demand increases because of union political influence, the present paper argues that the explanation can be different budgetary processes.

In contrast to the findings in OECD (1994), however, the intention of governments introducing decentralized bargaining has been to set local budgets prior to local bargains. But is commitment to a given budget credible? For the US, Derber (1988) describes large variation across state and local governments in budgetary processes relevant for collective bargaining. In the first attempt to practice cash limits in Sweden, the budget was adjusted upwards after the wage bargaining because the wage increased much more than the initial growth in the budget (see Holmlund, 1997). Since 1993, however, there has been a system of fixed nominal expenditures for wage costs and non-labor expenditures. These facts demonstrate that an analysis of public sector collective bargaining requires a discussion of different bargaining institutions. Thus, this paper compares the outcomes under different regimes, and thereby may indicate which institutional settings that are most likely to occur in different decision-making environments. However, this can only give a partial answer to the question of why institutions vary. In my view, the new literature on political strength may contribute to the

\[2\] There can be many possible advantages of more decentralized decision-making, summarized in OECD (1994, p. 37) as “promoting a more efficient and more responsive public service”, which are not discussed in the present paper.
understanding of institutional variation. Empirical evidence indicates that public deficits and public spending are negatively related to the strength of the political leadership. Several possible explanations are given in the literature. Political management in coalition governments may involve several difficulties (Roubini and Sachs, 1989), strong political parties and presidents may prevent universalistic behavior of legislators (Inman and Fitts, 1990), and a unified parliament may have an advantage in handling interest group pressure (Falch and Rattsø, 1997, 1999). In addition, political strength may influence the possibility to make credible budgetary commitments. Hard budget constraints are more likely under strong political leaderships than under weak political leaderships. Thus, the type of government may influence the bargaining institution at the local level, and a successful decentralization of collective bargaining may require a political leadership that is sufficiently strong to withstand pressure for increased budgets.\(^3\)

Section 2 presents the structure of the model and an overview of the bargaining regimes discussed. In the model, a sponsoring institution determines the budget of a bureau employing unionized workers. The employees’ union bargains either with the leadership of the bureau or directly with the sponsor. I term this local and central bargaining, respectively. Section 3 makes three different assumptions about the sequence of events under local bargaining. Central bargaining is discussed in Section 4, while Section 5 compares the utility levels of the sponsor and the union across the cases discussed. The ordering of the regimes based on utility levels depends on the parameter values in the model. A small numerical example illustrates the role of the most important parameters. Section 6 provides some concluding comments.

2. The model

The model includes three agents: the leadership of a public bureau producing a publicly provided good \(X\), a sponsoring institution deciding the budget \(B\) of the bureau, and a trade union covering all bureau employees.

2.1. Public sector preferences and budget constraints

Niskanen (1971) initiated the literature on strategic interaction between a sponsoring institution and public bureaus. Niskanen only considered budget maximizing bureaus

\(^3\) The consequences of commitment to a given policy are especially well described in the literature on central bank independence. An independent central bank with a conservative central banker will mitigate the time consistency problem of monetary policy. In an interesting paper, Moser (1999) argues that a credibly commitment to an independent central bank is only possibly under some legislative structures. Legal independence is in general dependent on the legislators who can change the law. However, with checks and balances in the legislative decision-making, a law cannot be modified by a simple majority rule. Thus, an independent central bank has some discretion in its actions without provoking an override by the legislators. By the same argument, one would expect checks and balances to strengthen the government’s possibility to set hard budget constraints.
and a passive sponsor in the interaction with the bureaus. More recent contributions relax these assumptions (see, for example, Moene, 1986; Chan and Mestelman, 1988). First, when both agents behave strategically, the institutional setting in which the interaction takes place is uncertain. As the common approach in the bureaucracy literature, I consider various assumptions of the institutional setting and compare the outcome of the different regimes. Second, a more realistic utility function of the bureau includes the production level and discretionary profit. Discretionary profit is interpreted as all kind of non-productive activities, including over-employment and extra salary. While the traditional bureaucracy literature is vague about the adjustment inside the bureaus, this is explicitly modeled here as a game between two parties with power; the bureau leadership and the union. I assume that the union does not care about production as in Niskanen’s model, while the bureau leadership only has preferences over bureau production. This implies that the governments succeed in hiring bureau bosses who maximize the bureau production without lobbying for more money and without taking other parts of public sector activity into consideration. This assumption is the rationale for perfect information in the model. When the bureau leadership wants to serve the sponsor as well as possible, the leadership will also report the production function. The only reason for discrepancy from the sponsor’s first best outcome is union bargaining power.

The production function of the bureau is of the Cobb–Douglas type

\[ X = a_t L^\alpha Z^{(1 - \alpha)}. \quad (1) \]

There are two inputs; employment \( L \) and non-labor input \( Z \), and \( 0 < \alpha < 1 \). Even though publicly provided goods usually are labor intensive, other inputs as materials, computer technology and external expertise enter into the production process. Typically, labor costs account of 60–80% of current expenditures of public services. The formulation in Eq. (1) implies that it is possible to substitute employment by non-labor input in a way that keeps the production constant. This is realistic, but not important for the model. It is the possibility to reduce the non-labor expenditures to finance increased wage costs that is important for the model. This is necessary to have bargaining over both the wage and employment because the sponsor determines the budget size.

The budget of the bureau is

\[ B = wL + Z, \quad (2) \]

where \( w \) is the wage and the price of \( Z \) is normalized to unity without loss of generality. Assume that the utility of the bureau leadership \( M \) is concave in the production size, \( M = a_2 X^\beta \), where \( 0 < \beta < 1 \). Utilizing Eqs. (1) and (2), the utility can be written

\[ M = aL^{\alpha \beta} (B - wL)^{(1 - \alpha)\beta}, \quad (3) \]

where \( a = a_1^{\alpha} a_2 \) is a constant.

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4 The bureau is the sponsor’s agent. The assumption of perfect information, however, rules out the traditional principal-agent mechanisms, see for example Arrow (1986) for an overview of such models.

5 The determination of administration costs, including the salary of the bureau leadership, is not discussed.
The sponsor has broader preferences than the production in the bureau. The budget level of the bureau is decided in a tradeoff with other means. I assume for simplicity a Cobb–Douglas utility function over the two public sector services \( X \) and \( C \), and the utility elasticity of the bureau production is equal to \( \beta \) for both the sponsor and the bureau leadership,

\[
V = bX^\beta C^{(1-\beta)}.
\]  

(4)

The budget constraint of the sponsor reads

\[
Y = C + B,
\]

(5)

where the income \( Y \) is assumed to be exogenous and the price of \( C \) is set equal to unity without loss of generality.

2.2. Union preferences

I assume a single trade union in the bureau and that the union cannot pay the members unemployment pay. The union members not employed by the bureau earn a reservation wage \( r \). The union utility is represented by a simplified Stone–Geary utility function (see, for example, Pencavel, 1991)

\[
U = L(w - r)^\alpha.
\]  

(6)

\( U \) is interpreted as the union loss during a dispute. \( \alpha \leq 1 \) measures the concavity of the utility function with respect to the gap between the wage and the reservation wage. \( \alpha = 1 \) implies a so-called rent maximizing union.6

2.3. First best outcome for the sponsor

As a reference point, I present the solution of the model in the case where the union has no bargaining power. The wage is given by the outside labor market. This gives a traditional labor demand schedule. Maximizing the utility of the sponsor Eq. (4) with respect to \( C \), \( L \) and \( Z \), subjected to the overall budget constraint \( Y = C + wL + Z \), yields the labor demand schedule

\[
L = \frac{Y}{w}.
\]  

(7)

6 The assumption that the union does not care about the level of \( C \), included in the utility function of the sponsor, simplifies the analysis. However, including \( C \) in the preferences of the union will not alter the qualitative features of the model. To see this, consider two different cases. First, if the budget of the bureau is set prior to the bargains, the bargains will not alter \( C \). The union loss during a dispute will be as Eq. (6), and the outcome of the model is independent of whether the union has preferences over \( C \) or not. Second, if the budget of the bureau is set after the bargains, the sponsor cannot use \( C \), or \( B \), strategically. Including \( C \) in the utility function of the union will therefore not alter the structure of the game. However, the union loss during a dispute will be lower than in Eq. (6) if a dispute is a strike and \( C = Y \) during the dispute. Notice that this requires a utility function for the sponsor where \( V(X = 0, C) > 0 \).
Because of the Cobb–Douglas functional forms, the wage elasticity of labor demand is equal to \(-1\) as in a cash limit specification. In Fig. 1, \(D\) is the labor demand schedule. If \(r\) is equal to the competitive wage, \(L^b\) can be interpreted as the competitive solution. All other outcomes yield lower utility for the sponsor.

2.4. Bargaining regimes

The bargaining outcome depends on the timing of the sponsor’s budgetary decision. Consider first the case of local wage and employment determination without union influence. Then the bureau leadership can independently alter the wage and employment if the budget size changes. Therefore, even if the bureau leadership wants to exploit a favorable strategic position, it can be difficult to come into such a position. This suggests as a “realistic” model one in which the budget is determined prior to the wage and employment. However, with union bargaining, contracts are making wage and employment rigid. This can put the bureau into a leader position of the budgetary game. Even though the bureau leadership does not want to exploit a favorable strategic position, bargaining contracts can force the public sector decision-makers into a regime where the budget is decided after the bargains. The local level offers the sponsor a given combination of the wage and employment, and the sponsor is passive as in the original bureaucratic literature. This is a realistic description of situations with a weak sponsor, there is a soft budget constraint and large bureaucratic power. The case is denoted

Fig. 1. The labour demand schedule, contract curves in Regime \(LwB\) for different values of \(\sigma\) and the effect of reduced \(\gamma_c\) from \(\gamma_c = \gamma_L\).
Regime $LwB$, where the name of the regime is motivated by the sequence of events. In the first stage of the game, the union and the bureau leadership bargain over the employment level. In the second stage, they bargain over the wage, and in the third stage, the sponsor decides the budget of the bureau. Lastly, $Z$ and $C$ are determined residually.\footnote{It follows from the budget constraint (2) that when the wage and employment are decided, the budget decision is a decision of the level of the non-labor input $Z$. This does not imply that $Z$ is outside the control of the local agents because the sponsor’s reaction to changes in the wage and employment is known by the assumption of perfect information. Thus, when the union and the bureau leadership bargain, they know the consequences for $Z$.}

The second regime, Regime $LBw$, assumes that the sponsor decides the bureau budget between the employment and wage bargains. Both the wage and the budget usually change each year. The budgetary process is prior to the fiscal year, and if the wage increases during the fiscal year without supplementary grants, it is reasonable to view the sponsor to be in a leader position compared to the wage bargain. Compared to the previous regime, the sponsor behaves strategically and it is limited bureaucratic power. Section 4 shows that this regime may yield the highest utility level for the sponsor, and it may therefore reflect a politically strong sponsor. The last local bargaining regime discussed, Regime $BLw$, considers a hard budget constraint. The budget is decided prior to local bargaining as in Holmlund (1997). This is usually thought of as the ideal institutional setting for the governments.

Finally, I present one case of central bargaining without strategic interaction between the central and local level. For the sponsor, commitment to a given budget is likely to be harder when he decides the adjustment in the bureau compared to the situation where he only sets the budget. The survey in OECD (1994) shows that the governments in most countries with centralized bargaining, as for example France and Germany adjust the budgets to take collective agreements into account. In addition, the situation where the budget is decided at the latest stage of the game is the most interesting theoretical case. If the budget decision is made prior to bargaining, both the sponsor and the bureau leadership want to maximize bureau production for the given budget.

3. Local bargaining

In this section, the union bargains with the bureau leadership. The outcome is illustrated by the Nash bargaining solution. Even though this is an axiomatic approach, the solution can be interpreted as a result of bargaining with alternating offers (see Binmore et al., 1986).

When a dispute implies $L = 0$, Eq. (3) represents the loss during a dispute for the bureau leadership. The Nash-maximands read

$$
\Omega_i = \left[ L(w - r)^\sigma \right]^{\gamma_i} \left[ (L + B - wL)^{(1 - \sigma)\beta} \right]
$$

where $i = w, L$. \hfill (8)

$\gamma_w$ and $\gamma_L$ are the relative bargaining powers of the union in the wage and employment bargains, respectively. Thus, I allow for different bargaining power in the two bargains.
For the private sector, Manning (1987a) cites evidence for \( \gamma_c \neq \gamma_L \). In the public sector, trade unions in reality affect employment through several different channels and various political decisions, while the wage bargain is more formalized. There is no reason why \( \gamma_w = \gamma_L \).

### 3.1. Regime \( LwB \): Soft budget constraint

The outcome is derived by backward induction. In the third stage of the game, the sponsor maximizes the utility \( \gamma \) with respect to \( B \) and \( C \), subjected to the production function (1) and the budget constraint (5). The first order condition is \( (1 - \beta)Z = (1 - \alpha)\beta C \). Utilizing the budget constraints (2) and (5), the sponsor’s reaction-function can be written

\[
B = qY + (1 - q)wL \quad \text{where} \quad q = \frac{(1 - \alpha)\beta}{1 - \alpha\beta} < 1. \tag{9}
\]

The budget of the bureau is increasing in the wage and employment levels. In addition, more money is allocated to the bureau when the marginal utility of \( X \) (measured by \( \beta \)) increases or the productivity of \( L \) (measured by \( \alpha \)) decreases. In the latter case, the productivity of \( Z \) increases, and an increased budget has a larger effect on \( X \) for given \( w \) and \( L \).

The structure of the model implies that the bargaining parties take employment as given in the wage bargain, while they treat the wage as endogenous in the employment bargain. The first order conditions for the maximization of Eq. (8) with respect to \( w \) and \( L \) are

\[
(1 - \alpha)\beta(w - r) \left( L - \frac{dB}{dw} \right) = \gamma_w\alpha Z, \tag{10}
\]

\[
(1 - \alpha)\beta wL = (\alpha\beta + \gamma_L)Z + (1 - \alpha)\beta L \frac{dB}{dL} + \left( \gamma_L\alpha \frac{Z}{w - r} - (1 - \alpha)\beta L \right) L \frac{dw}{dL}. \tag{11}
\]

To illustrate the solution of the model, it is useful to discuss the case when \( \gamma_w = \gamma_L \). Lemma 1 states that the outcome in this case is similar to simultaneous bargaining over \( w \) and \( L \).

**Lemma 1.** The model is the efficient bargaining model if \( \gamma_w = \gamma_L \) (Manning, 1987a, Proposition 1).

**Proof.** When \( \gamma_c = \gamma_L \), the two bargains have the same objective function. Lemma 1 follows from the envelope theorem. \( \square \)

In the efficient bargaining model, the outcome is on the contract curve describing bilaterally optimal combinations of the wage and employment for the bargaining parties.
There is no opportunity to increase one party’s utility without reducing the other party’s utility. The relative bargaining power determines the actual outcome on the contract curve. The following result implies that the slope of the contract curve differs from the case when the employer maximizes profit.

**Proposition 1.** With efficient bargaining at the local level and a soft budget constraint

(i) the contract curve reads

\[ L = \frac{\alpha \sigma Y}{(\alpha + \sigma - 1)w + (1 - \alpha)z}; \]  

(ii) the contract curve has a negative slope if \( \sigma > (1 - \alpha) = \tilde{\sigma}^{-}\);

(iii) the contract curve has a positive slope if \( \sigma < (1 - \alpha) = \tilde{\sigma}^{+}\).

The proof of this and all subsequent propositions are in Appendix A.

As in efficiency bargaining models for the private sector, the shape of the contract curve depends on the union utility function. For \( \sigma = 1 \), the contract curve in a profit maximizing firm will be vertical in the \( w-L \) space (see, for example, Johnson, 1990; Pencavel, 1991; de la Croix and Toulemonde, 1995). In a production maximizing public bureau, however, the contract curve is negatively sloped in this case. This is because the public employer is faced with a budget constraint. The loss of increased wage for given employment level is higher for an employer with a budget constraint than for a profit maximizing employer because \( Z \) must be reduced. The deviation from the optimal relationship between the inputs increases. Generally, if the union utility function is the same in both sectors, \( dL/dw \) is smaller along the contract curve in the public sector than in private firms.

Fig. 1 shows the contract curve for three different values of \( \sigma \). For \( w = r \), the employment is higher than in the sponsor’s first best outcome because the sponsor pays a part of the cost of increased employment. The marginal cost of employment for the bureau leadership is less than \( w \) when \( dB/dL > 0 \). This is the first mover advantage. It also follows that when \( \sigma = \tilde{\sigma}^{-} \) the employment level is always at the efficient level from the bureau leadership’s point of view in the sense that the relationship between the marginal utilities of \( L \) and \( Z \), taking into account the effect of \( L \) on \( B \), is equal to \( r \).

The outcome will not be on the contract curve when \( \gamma_{u} \neq \gamma_{L} \). Proposition 2 describes the actual outcome.

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8 McDonald and Solow (1981) specify the union loss during a dispute as \( L[u(w) - u(r)] \), where \( u \) is a concave utility function. With this formulation, the marginal utility of \( w \) is positive and decreasing. Efficient bargaining with a profit-maximizing firm therefore yields a positively sloped contract curve. Utilizing this union utility function in the present setup gives an ambiguous sign of the slope of the contract curve. Thus, one needs more structure on the union utility function to determine the sign of the slope. In a cash limit framework, Leslie (1985) develops a contract curve in the case of a profit-maximizing employer, reproducing the McDonald and Solow (1981) result. In a model with general functional forms in a public sector institutional setting, Inman (1981) presents an upward sloping contract curve, but without calculations.
Proposition 2. With local bargaining and a soft budget constraint

(i) the wage and employment is

\[ w = (1 + m^i) r \]  
where \[ m^i = \frac{\gamma_w \sigma}{\alpha \beta + \gamma_L (1 - \sigma)} \left( 1 - \alpha \right) \beta + \gamma_L \sigma \]. \hfill (13)

\[ L = \frac{\alpha \beta + \gamma_L (1 - \sigma)}{\beta + \gamma_L} \frac{Y}{r}; \hfill (14) \]

(ii) \( \frac{\partial L}{\partial \gamma_w} = 0 \) and \( \frac{\partial w}{\partial \gamma_w} > 0; \)

(iii) if \( \sigma > \tilde{\sigma}^i \), \( \frac{\partial L}{\partial \gamma_L} < 0 \) and \( \frac{\partial w}{\partial \gamma_L} > 0; \)

(iv) if \( \sigma < \tilde{\sigma}^i \), \( \frac{\partial L}{\partial \gamma_L} > 0 \) and \( \frac{\partial w}{\partial \gamma_L} < 0. \)

A rise in the relative union bargaining power in the wage bargain \( \gamma_w \) has the standard effect; the wage increases. When employment is given, the union has nothing to lose from increased wage. However, \( \gamma_w \) does not influence the employment level even though the effect on the wage is taken into account in the employment bargain (see also Manning, 1987a). This is because the optimal employment level of both the union and the bureau leadership is independent of \( \gamma_w \).

The effect of the relative union bargaining power over employment follows directly from the slope of the contract curve. If \( \sigma \) is large, the union wants lower employment than the bureau leadership in order to achieve a higher wage. In fact, the union fights for low employment in the employment bargain. A common objection to the efficient bargaining model is that unions lack the ability to influence employment. \( \gamma_L = 0 \) is a special case of the model. Fig. 1 evaluates the effect of reducing \( \gamma_L \) starting from \( \gamma_u = \gamma_L \). When \( \sigma > \tilde{\sigma}^i \), reduced \( \gamma_L \) increases the employment level and reduces the wage. The opposite is true when \( \sigma < \tilde{\sigma}^i \). When \( \sigma = \tilde{\sigma}^i \), \( \gamma_L \) has no effect on the outcome; the optimal employment level is equal for the union and the bureau leadership.

Empirical evidence for the US indicates that the employment level in local public sector services is higher in units with union bargaining compared to non-bargaining units, see for example Inman (1981), Zax and Ichniowski (1988), and the survey in Freeman (1986). While Inman interprets this result as evidence of an upward sloping contract curve, Freeman and Zax and Ichniowski argue that union political influence increases labor demand. The present model shows that bargaining rights itself can change the possible wage-employment combinations. Even in a situation with a negatively sloped contract curve, the employment level can be above the first best outcome.

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9 Increased employment has two opposite effects on both the utility levels of the union and the bureau. It is a positive direct effect on both utility levels, while the negative effects go through reduced \( w \) and \( Z \), respectively.

10 Some papers argue that this result can be due to misspecification. Trejo (1991) argues that the effect is at least partly related to higher unionization probability in large units. In Valletta (1993), the significance disappears when he controls for potential fixed effects. However, Valletta uses data for only two years with few changes in bargaining status, which makes it difficult to identify possible union effects.
for the sponsor. If the competitive wage equals the reservation wage and \( \sigma = 1 \), this is the case if \( \gamma_L < (1 - \beta) \). In accordance with \( \sigma > \sigma^* \) and \( \gamma_L > 0 \), Zax and Ichniowski find lower employment in bargaining units than in units without bargaining when they control for the budget size.

It is interesting to compare the results derived above with the bureaucracy literature. In Moene’s (1986) model, there is always over-supply when the bureau moves first. When discretionary profit is union rent as in the present paper, the result depends on the size of the union bargaining powers. When \( \gamma_u = \gamma_L = 0 \), the outcome is optimal for the bureau leadership, and there is over-supply as in Moene. The bureau extracts a higher budget by setting a non-optimal combination of the inputs from the sponsor’s point of view. However, with some union influence, the production becomes expensive because the wage is high, and the bureau under-supplies.

3.2. Regime LBw: Budget decided between the bargains

Compared to the previous regime, the sponsor now behaves strategically with respect to the wage bargain. The present setup extends the sequential bargaining framework of Manning (1987a,b) by explicitly modeling the difference between the objective functions in the two bargains. The wage level is determined by Eq. (10) and \( d B / d w = 0 \). The wage is increasing in the budget size because the bargaining parties have more to share. Increased employment has a negative effect on the wage because, for the bureau leadership, a wage increase then becomes more costly at the margin. In the second stage of the game, the sponsor maximizes utility (4) taking into account the wage reaction in the third stage. The first order condition is \( (1 - \beta) Z = (1 - \alpha) \beta (1 - L dw / d B) C \).

Utilizing the budget constraints (2) and (5) and the wage reaction-function (10), the sponsor’s reaction-function can be written

\[
B = q Y + (1 - q) r L. \quad (15)
\]

Compared to Regime LwB, the budget is lower (when \( w > r \), i.e., \( \gamma_u > 0 \)). The sponsor takes into account that only a part of an increased budget ends up in increased \( Z \). The employment level is determined by Eq. (11).

Proposition 3. (i) When the budget is determined in the second stage of the game (Regime LBw), the wage is

\[
w = (1 + q m) r. \quad (16)
\]

(ii) When employment is determined in the first stage of the game (Regime LwB and LBw), the employment level is independent of the timing of the rest of the game.

\[\text{11} \]See also the numerical example in Section 5.

\[\text{12} \]With monopolistic price formation, de la Croix and Toulemonde (1995) prove that a situation where the firm produces less with more workers may Pareto-dominate the efficient bargaining outcome because the marginal revenue of the firm may be negative due to the bargaining. Reduced production may increase the revenues. They speculate about whether this can explain over-manning in public firms. In the present model, this is not possible because the marginal utility of production is always positive.
When the sponsor decides the budget between the two bargains, increasing employment extracts a higher budget while increasing the wage does not. Compared to Regime \( LwB \), the cost of increasing employment for the bureau leadership is lower than the cost of increasing the wage. Employment is raised initially because this is a way of increasing the total budget. Thus, the employment objective gets a relatively greater weight in the interaction between the union and the bureau leadership, and the wage is squeezed subsequently. The result of equal employment levels in Regime \( LwB \) and \( LBw \) is due to the Cobb–Douglas production function. With more general functional forms, it is ambiguous in which regime the employment level is highest. However, in the present model, the sponsor is clearly better off if he commits to a given budget before the wage bargain since the wage is low in Regime \( LBw \). This is also reflected in the shape of the contract curves. They are illustrated as CC\(_{LwB}\) and CC\(_{LBw}\) in Fig. 2. Proposition 3 implies that the contract curve is flatter in the present regime than in Regime \( LwB \).

Since both the budget and employment are given when wage bargaining takes place, there is not much room for wage increases. If this situation is common, it can be a possible explanation of a lower union–nonunion wage mark-up in the public sector than in the private sector. To be more precise, however, private sector bargaining needs to be modeled.

3.3. Regime \( BLw \): Hard budget constraint

The bargaining outcome is described by Eqs. (10) and (11) and \( dB/dw = dB/dL = 0 \). Comparing the outcomes in the different regimes yields the following general results.

Fig. 2. Contract curves in different regimes when \( \sigma^l < \sigma < \sigma^c \).
Proposition 4. With local bargaining

(i) the sign of the slope of the contract curve is independent of the timing of the budget decision;
(ii) the wage is independent of the timing of the budget decision when nothing happens between the wage and employment bargains (Regime LwB and BLw);
(iii) with a hard budget constraint (Regime BLw), the employment level is lower than in the other regimes (Regime LwB and LBw).

It is the possibility of substituting other factors for employment that determines the slope of the contract curve, and not the size of the budget or the form of the sponsor’s reaction–function. This is further discussed in the Section 4. The result of equal wage in Regime LwB and BLw is due to the Cobb–Douglas specification of the production function. The difference between these regimes is the employment levels. The outcome will be the first best outcome for the sponsor in Regime BLw if the union has no bargaining power because the sponsor has the first mover advantage. CC_{BLw} is to the left of CC_{LwB} and CC_{LBw} in Fig. 2. Thus, the employment level is always lowest in Regime BLw.

4. Central bargaining

The degree of decentralization of wage determination varies across countries, see for example Calmfors and Driffill (1988) for a discussion mainly related to the private sector and OECD (1994, 1997) for a discussion of the public sector. Is it possible for the sponsor to increase his utility by bargaining directly with the union? While the bureau leadership is only concerned about the input mix, the sponsor is also concerned about the relationship between different public sector services. The main difference between local and central bargaining in the present model and Strøm (1999) is the specification of the sponsor’s utility function. Strøm specifies the utility of the national government (i.e., the sponsor) as the sum of the utilities of the local governments (i.e.,

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13 This is the same result as in efficient bargaining models for the private sector. McDonald and Solow (1981, p. 908), using more general functional forms, cannot sign the wage effect of changed real product demand. When an exogenous shock reduces the employment level, the union loses and the employer gains because the outcome is outside the labor demand function. The adjustment to the reduced employment will transfer some of the employer’s gain to the union, where “the part of this adjustment that falls (efficiently) on wages will involve an increase in the wage”.

14 One extreme example is primary school teachers in Norway. They are employed by local governments, which in Norway cannot influence their budget size. The central government is therefore the sponsoring institution of a local government. The teacher union has for a long period bargained over wages, working time, and working rules exclusively with the national government, see the discussion of the Norwegian experience in Falch and Rattsø (1997). For the other Scandinavian countries, this was the case up to the late 1980s. Thereafter, some decisions regarding pay and working conditions are decentralized. In addition, the local governments in these countries have some flexibility in tax issues.
the bureaus). Then central bargaining with cash limits yields exactly the same outcome as local bargaining with local financing.

To focus on the role of bargaining level, I only consider the extreme case when the union and the sponsor bargain over both the wage and employment. The model can be viewed as if the sponsor does not use a bureau to supply the public service because the only role of the bureau is to set the non-labor input residually. Since there is no strategic interaction between the central and local level, I focus on the case where the sponsor adjusts the budget to take collective agreements into account.

4.1. Regime C: Central bargaining

The Nash-maximands with central bargaining read

$$\Omega_i = \left[ L(w-r)^\sigma \right] Y \left[ b(aL^\alpha (B-wL)^{(1-a)} b(Y-B)^{(1-\beta)} \right]$$

where \( i = w, L \).

Maximizing Eq. (17) with respect to the wage and employment, treating the wage as endogenous in the employment bargain, gives the first order conditions

$$\left(1 - \alpha \right) \beta (w-r) L = \gamma s, \sigma Z,$$  \hspace{1cm} (18)

$$\left(1 - \alpha \right) \beta wL = (\alpha \beta + \gamma s)Z + \left( \gamma L, \sigma \frac{Z}{w-r} - (1-\alpha) \beta L \right) \frac{dw}{dL}. \hspace{1cm} (19)$$

Local and central bargaining differ in two important ways. First, the sponsor has broader preferences than the bureau leadership. Second, the hierarchical structure is without interest. How the wage and employment affect the budget size does not influence the bargains due to the envelope theorem. This regime therefore bears a close resemblance to Regime BLw. To see the differences between the regimes, it is useful to compare the contract curves.

Proposition 5. With central bargaining and a soft budget constraint

(i) the contract curve reads

$$L = \frac{\alpha \beta \sigma Y}{(\alpha \beta + \sigma - 1) w + (2 - \alpha - \beta) r}; \hspace{1cm} (20)$$

(ii) the contract curve has a negative slope if \( \sigma > (1 - \alpha \beta) = \tilde{\sigma}^{-}; \)

(iii) the contract curve has a positive slope if \( \sigma < (1 - \alpha \beta) = \tilde{\sigma}^{+}. \)

Because \( \tilde{\sigma}^{-} > \tilde{\sigma}^{+}, \frac{dL}{dw} \) is larger along the contract curve with central bargaining than with local bargaining. While the bureau only can substitute \( L \) for \( Z \), the sponsor can substitute \( L \) for both \( Z \) and \( C \). In general, when the wage costs become a smaller part of the budget, the contract curve moves as from CC_{BLw} to CC_{C} in Fig. 2. There are two possible sources behind this shift. One possibility is that more factors can be
substituted for $L$. This is the difference between local and central bargaining in the present model. Another possibility is reduced productivity of $L$ relative to $Z$, i.e., a lower $\alpha$. Both cases are easy to see by inspection of $\hat{\sigma}^l$ and $\hat{\sigma}^c$.15

Regarding the actual outcome, the following results emerge.

Proposition 6. (i) With central bargaining and a soft budget constraint, the wage and employment is

$$w = (1 + m^c) r$$

where

$$m^c = \frac{\gamma_w \sigma}{\alpha \beta + \frac{\gamma_L \sigma}{\alpha \beta + \gamma_L (1 - \sigma)} 1 - \alpha \beta + \gamma_L \sigma},$$

(21)

$$L = \frac{\alpha \beta + \gamma_L (1 - \sigma) }{1 + \gamma_L} \frac{Y}{r}.$$  

(22)

(ii) The wage is higher with central bargaining than with local bargaining if $\gamma_w > \gamma_L$.

To fix intuition, it is useful to compare the present regime and Regime $BL_w$ in the case when $\gamma_L = 0$. In both cases, the bureau cannot exploit a favorable strategic position and $L = L^B$. However, the utility loss of increased wage is lower for the sponsor in Regime $C$ than for the bureau leadership in Regime $BL_w$ because the sponsor can change $C$ in addition to $Z$ as a response to the higher wage. Thus, the wage is higher with central bargaining. When $\gamma_L$ increases from zero, this effect also leads to higher employment, partially reducing the wage advantage for the union of central bargaining. If $\gamma_w = \gamma_L$, the wage is independent of the bargaining level. This is the rigid wage result again.16

5. Comparing the utility levels across regimes

This section compares the utility levels of the sponsor and the union across the regimes discussed. Proposition 7 presents the general results while Table 1 presents a numerical version of the model to through more light on the importance of the size of some parameters.

Proposition 7. Denoting the different regimes with lowered letters, for $\gamma_w > 0$

(i) $V_{L_BW} < V_{LB_w}$ and $U_{L_BW} > U_{LB_w}$;

(ii) if $\gamma_w \geq \gamma_L$, $V_{BL_w} > V_C$ and $U_{BL_w} < U_C$;

---

15 Consider two extreme cases when $\sigma = 1$. When the objective function of the public sector employer only includes employment ($\alpha = \beta = 1$), there is no possibility to substitute employment with other factors, and the contract curve will coincide with the labor demand schedule. On the other hand, with profit maximization, there is no budget constraint at all and the contract curve is vertical.

16 Notice that there is also a close resemblance between the present regime and Regime $LwB$. There is a soft budget constraint in both cases, and if $\gamma_w \to \infty$, the union will extract the whole rent available, i.e., $wL = Y$. In both regimes, $X \to 0$ in this case. However, the employment is higher in Regime $LwB$ than in Regime $C$ for finite values of $\gamma_L$ because the local level has the first mover advantage in Regime $LwB$. 
(iii) for the sponsor, there exists a value of \( \sigma \), denoted \( \tilde{\sigma}^+ \), for which \( V_{LBw} > V_{BLw} \) if \( \sigma > \tilde{\sigma}^+ \) and \( V_{LBw} < V_{BLw} \) if \( \sigma < \tilde{\sigma}^+ \); 
(iv) for the union, there exists for some combinations of \( \gamma_u \) and \( \gamma_L \), and \( \gamma_u > \gamma_L \), a value of \( \sigma \), denoted \( \tilde{\sigma}^+ \), for which \( U_{LwB} > U_C \) if \( \sigma < \tilde{\sigma}^+ \) and \( U_{LwB} < U_C \) if \( \sigma > \tilde{\sigma}^+ \).

The lower wage in Regime LBw than in Regime LwB yields (i) since there are equal employment levels. The form of the contract curves and Proposition 6 give the intuition of part (ii). The contract curve with central bargaining is to the right of the contract curve in Regime BLw. Thus, \( V_{BLw} > V_C \) and \( U_{BLw} < U_C \) if the wage is not lower in Regime C than in Regime BLw. A sufficient condition is that \( \gamma_u \geq \gamma_L \).

Parts (iii) and (iv) of Proposition 7 exploit how \( \sigma \) influences which of the regimes that yield the highest utility for the sponsor and the union, respectively, when \( \gamma_u \geq \gamma_L \). When \( \gamma_u \) is “much smaller” than \( \gamma_L \), the results can be different because part (ii) of the proposition changes. However, the case when \( \gamma_u \geq \gamma_L \) is most interesting because it covers both the efficient bargaining model and the case when \( \gamma_L = 0 \). If \( \sigma \) is large, the outcome is a high wage. In this situation, the sponsor prefers a regime that yields a relatively low wage. This is the case in Regime LBw. When \( \sigma \) is small, the outcome is a high employment level. Regime BLw is now favorable because it yields a relatively low employment level. The size of the critical value of \( \sigma \), \( \tilde{\sigma}^+ \), depends on the parameter values involved. Appendix A proves that \( \tilde{\sigma}^+ \) is independent of \( \gamma_u \) and that \( 0 < \tilde{\sigma}^+ < 1 \) if \( \gamma_L \) is not “too small”. For the union, Regime C can be preferable because the wage in this regime can be high. A high wage is most valuable for the union when \( \sigma \) is large.

The value of \( \sigma \) and the difference between \( \gamma_u \) and \( \gamma_L \) are important for the preference orderings of the regimes. Table 1 presents the parameter values used in the numerical version of the model. Regarding union preferences, the rent maximizing union utility function (\( \sigma = 1 \)) is popular in theoretical models. However, the small empirical literature that exists on the form of union utility functions suggests that union preferences are more heavily weighted toward employment than implied by the rent maximizing specification, see Clark and Oswald (1993) and the survey in Pencavel (1991). I therefore offer simulations for both \( \sigma = 1 \) and \( \sigma < 1 \). To illustrate the effect of different bargaining powers in the wage and employment bargains, I use both \( \gamma_u = \gamma_L > 0 \) and \( \gamma_u > \gamma_L = 0 \).

When \( \gamma_u = \gamma_L = \sigma = 1 \), Example I in Table 1, \( V_{LBw} > V_{BLw} \). In this case, local bargaining and a hard budget constraint will not occur. It is optimal for the sponsor to decide the budget after the realization of the employment level. When \( \sigma \) is reduced from 1 to 0.6 (notice that \( \tilde{\sigma}^1 < \sigma = 0.6 < \tilde{\sigma}^+ \)), the optimal regime for the sponsor shifts from Regime LBw to Regime BLw. This implies that \( 0.6 < \tilde{\sigma}^+ < 1 \).\(^{17}\)

\(^{17}\) In Examples I and II in Table 1 (\( \gamma _u = 1 \)), \( \tilde{\sigma}^+ = 0.72 \). From Proposition 7, it follows that Regime LBw yields highest utility for the sponsor in Example I (\( \sigma = 1 \)) and Regime C yields highest utility in Example II (\( \sigma = 0.6 \)). For the union, Regime LwB yields highest utility both in Examples I and II (\( \tilde{\sigma}^+ \) is not defined). Regarding Example III (\( \gamma_u = 0 \)), it is possible to show that \( \tilde{\sigma}^+ = 0.93 \), and \( \gamma_L = 0 \) implies that \( \tilde{\sigma}^+ \rightarrow \infty \). \( \tilde{\sigma}^+ = 1 \) when \( \gamma_L = 0.05 \) and \( \tilde{\sigma}^1 = 1 \) when \( \gamma_L = 0.21 \).
Table 1
Simulated outcomes in the different regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>Wage ($w$)</th>
<th>Employment ($L$)</th>
<th>Production ($X$)</th>
<th>Sponsor utility ($V$)</th>
<th>Union utility ($U$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>I</td>
<td>II</td>
<td>III</td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td></td>
<td>$\gamma_L = 1$, $s = 1$</td>
<td>$\gamma_L = 1$, $s = 0.6$</td>
<td>$\gamma_L = 0$, $s = 0.6$</td>
<td>$\gamma_L = 1$, $s = 0.6$</td>
<td>$\gamma_L = 1$, $s = 0.6$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>I</td>
<td>II</td>
</tr>
<tr>
<td>$L_wB$</td>
<td>5.00</td>
<td>1.92</td>
<td>1.71</td>
<td>50.0</td>
<td>15.7</td>
</tr>
<tr>
<td></td>
<td>2.33</td>
<td>1.31</td>
<td>1.24</td>
<td>50.0</td>
<td>15.7</td>
</tr>
<tr>
<td>$BwL$</td>
<td>5.00</td>
<td>1.92</td>
<td>1.71</td>
<td>25.0</td>
<td>13.6</td>
</tr>
<tr>
<td></td>
<td>2.33</td>
<td>1.31</td>
<td>1.24</td>
<td>25.0</td>
<td>13.6</td>
</tr>
<tr>
<td>$C$</td>
<td>5.00</td>
<td>1.92</td>
<td>2.33</td>
<td>25.0</td>
<td>18.6</td>
</tr>
<tr>
<td></td>
<td>50.0</td>
<td>31.0</td>
<td>29.7</td>
<td>50.0</td>
<td>31.0</td>
</tr>
</tbody>
</table>

Note: The parameter values are $a = b = \gamma_w - r = 1$, $\alpha = \beta = 0.5$, and $Y = 100$. If $r$ is equal to the competitive wage, the first best outcome for the sponsor is $L = X = 25.0$ and $V = 35.4$. 
Reduced union bargaining power over employment reduces the utility of the union while the utility of the other bargaining party increases. The union influence is reduced. Notice, however, that the utility of the sponsor is positively correlated with $\gamma_k$ in Regimes $LwB$ and $LBw$. This is true as long as the contract curves have negative slopes. A higher $\gamma_k$ reduces the budget of the bureau while the production is almost unchanged.

Many recent theoretical contributions discuss how economic performance differs between centralized and decentralized decision-making systems, see for example Moene et al. (1993) for a wage bargaining model and Caillaud et al. (1996) for a moral hazard model with bargaining at the local level. A likely situation is that the central level can influence the level at which collective bargaining take place. One interesting question is therefore whether the sponsor prefers local or central bargaining. The answer depends on the possibility to commit to a given budget. If commitment to a hard budget constraint is always possible, the bargaining level has no impact on the outcome. However, because the sponsor does not take part in local bargaining, the ability to commit to a hard budget constraint seems most likely with decentralized bargaining. One interesting case is when the sponsor is able to commit to a given budget before the wage bargain under local bargaining and cannot commit at all under central bargaining. Example III of the simulation shows that there is no simple conclusion on whether local or central bargaining is optimal in this case. The optimal bargaining level depends on both the parameter values of the model and the budget commitment abilities.

6. Conclusion

This paper suggests that simply applying the results from union bargaining models developed for private firms when analyzing labor market outcomes in the public sector may be misleading. Budget constraints and budgetary processes are important factors of collective bargaining in the public sector. In organizations with a hierarchical structure, union influence can put the local level in a leader position in the sense that the sponsor determines the local budget after local decisions. This is a favorable position for the union. The analysis shows that union influence therefore can lead to both a high wage and a high employment level even in the case of a negatively sloped contract curve.

The degree of decentralization of collective bargaining differs across countries. Two relevant factors for decisions over bargaining level follows from the model in this paper. Consider the arguments for local bargaining from the sponsor’s point of view. If union bargaining power over the wage is relatively high compared to union bargaining power over employment, local bargaining yields a relatively low wage. In addition, if the possibility for the sponsor to commit to a given budget is higher with local bargaining, then this also makes local bargaining more attractive for the sponsor. When the union utility is weighted towards wages, the sponsor prefers an institutional setting that gives little room for wage increases. This is the case when the budget decision splits local bargains over the wage and employment.

The model assumes stable and well-defined utility functions for the well-informed public sector decision-makers. While this highlights important differences between
collective bargaining in the public and private sectors, it rules out the role of politics and uncertainty. To assess more precisely the public sector labor markets, collective bargaining models should in the future be merged with public choice models and principal-agent models. Hopefully, such models could give further insight into factors that are important for the determination of collective bargaining institutions.

Acknowledgements

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Appendix A

Proof of Proposition 1. From Eq. (9), it follows that $dB/dL = (1 - q)(w + Ldw/dL)$ and $dB/dw = (1 - q)L$. Utilizing these results and plugging the value of $\gamma_u = \gamma_L$ from Eq. (10) into Eq. (11) give

$$q(1 - \alpha)\sigma wL = \alpha\sigma Z + q(1 - \alpha)(w - r)L.$$  \hfill (A1)

Plugging Eq. (9) into Eq. (2) gives

$$Z = B - wL = q(Y - wL).$$ \hfill (A2)

Plugging Eq. (A2) into Eq. (A1) yields Eq. (12). Parts (ii) and (iii) follow from inspection of Eq. (12). \hfill \Box

Proof of Proposition 2. The value of $L$ is found as follows. From Eqs. (10) and (A2) it follows that

$$wL = D^I[(1 - \alpha)\beta rL + \gamma_u \sigma Y]$$ \hfill (A3)

$$-\frac{dw}{dL} = -D^I\gamma_u \sigma \frac{Y}{L^2}.$$ \hfill (A4)

Plugging Eqs. (10), (A2), (A3), (A4), and the partial derivatives of $B$ from Eq. (9) into Eq. (11) yields after some manipulation the employment as Eq. (14).

The wage is found by plugging Eq. (14) into Eq. (A3).

$$[\alpha\beta + \gamma_L(1 - \sigma)]w = D^I(1 - \alpha)\beta \left[\alpha\beta + \gamma_L(1 - \sigma)\right]r$$

$$+ D^I(\beta + \gamma_L)\gamma_u \sigma r.$$ \hfill (A5)
Inserting the value of \( D^1 \) yields after some manipulation
\[
\left[ \alpha \beta + \gamma_L(1 - \sigma) \right] \left[ (1 - \alpha) \beta + \gamma_L \sigma \right] w \\
= \left[ \alpha \beta + \gamma_L(1 - \sigma) \right] \left[ (1 - \alpha) \beta + \gamma_L \sigma \right] r \\
+ \left[ (1 - \alpha) \beta + \gamma_L \sigma \right] \gamma_L \sigma r. 
\] (A6)

This is equal to Eq. (13). Parts (ii), (iii), and (iv) follow from inspection of Eqs. (13) and (14). \( \square \)

**Proof of Proposition 3.** The first order conditions for the determination of the wage and employment levels are given by Eqs. (10) and (11), respectively, and \( dB/dw = 0 \). From Eqs. (10) and (2), it follows that
\[
wL = D^1 \left[ (1 - \alpha) \beta rL + \gamma_L \sigma B \right]. 
\] (A7)
\[
\frac{dw}{dL} = D^1 \gamma_L \sigma \left( \frac{1}{L} \frac{dB}{dL} - \frac{B}{L^2} \right). 
\] (A8)

Plugging Eq. (15) into Eq. (2) gives
\[
Z = B - wL = qY + (1 - q) rL - wL. 
\] (A9)

Plugging Eqs. (10), (A7), (A8) and (A9) into Eq. (11) and utilizing Eq. (15) yield Eq. (14). This proves part (ii).

The wage is found by plugging Eqs. (14) and (15) into Eq. (A7).
\[
\left[ \alpha \beta + \gamma_L(1 - \sigma) \right] w = D^1 \left[ (1 - \alpha) \beta + (1 - q) \gamma_L \sigma \right] \\
\times \left[ \alpha \beta + \gamma_L(1 - \sigma) \right] r + qD^1 \left( \beta + \gamma_L \right) \gamma_L \sigma r. 
\] (A10)

Manipulations along the line of the Proof of Proposition 2 yield Eq. (16). This proves part (i). \( \square \)

**Proof of Proposition 4.** The first order conditions for the determination of the wage and employment levels are given by Eqs. (10) and (11) and \( dB/dL = dB/dw = 0 \). Plugging Eqs. (10), (A7) and (A8) into Eq. (11) gives
\[
L = \frac{\alpha \beta + \gamma_L(1 - \sigma) \frac{B}{\beta + \gamma_L}}{r}. 
\] (A11)

Comparison of Eqs. (A11) and (14) proves part (iii) since \( B < Y \).

Plugging Eq. (A11) into Eq. (A7) yields Eq. (A5). This proves part (ii). The contract curve is found by elimination of \( \gamma_L \) from Eqs. (10) and (11) along the line of the Proof of Proposition 1.
\[
L = \frac{\alpha \sigma B}{(\alpha + \sigma - 1) w + (1 - \alpha) r}. 
\] (A12)

Proposition 3 and comparison of Eqs. (12) and (A12) prove part (i). \( \square \)
Proof of Proposition 5. Plugging Eq. (A2) and the value of $\gamma_n = \gamma_L$ from Eq. (18) into Eq. (19) yields Eq. (20). Parts (ii) and (iii) follow from inspection of Eq. (20).

Proof of Proposition 6. From Eqs. (18) and (A2), it follows that
\[
wL = D^*\left[(1 - \alpha) \beta rL + q\gamma_n Y\right] \quad \text{where} \quad D^* = \left[(1 - \alpha) \beta + q\gamma_n \sigma \right]^{-1}.
\]
\[
\frac{dw}{dL} = -qD^*\gamma_n \sigma \frac{Y}{L^2}.
\]  
Plugging Eqs. (18), (A2), (A13) and (A14) into Eq. (19) yields the employment as Eq. (20). Combining Eqs. (22) and (A13) as in the Proof of Proposition 2 gives the wage as Eq. (21).

Comparing the wage in Regime C (Eq. (21)) with the wage in Regime $LwB$ and $BLW$ (Eq. (13)), it follows that the wage in Regime C is highest if
\[
\frac{1 - \alpha \beta + \gamma_L \sigma}{1 - \alpha \beta + \gamma_n \sigma} > \frac{(1 - \alpha) \beta + \gamma_L \sigma}{(1 - \alpha) \beta + \gamma_n \sigma}.
\]
(A15)
This is fulfilled if $\gamma_n > \gamma_L$, and ends the proof.

Proof of Proposition 7. Let lowered letters denote regime. First, I prove part (i). Eqs. (A2) and (A9) yield that
\[
Z_{LwB} - Z_{LBw} = L_{LBw} - (1 - q) rL_{LBw} - qw_{LBw} L_{LBw}.
\]
Proposition 3 says that $L_{LBw} = L_{LBw}$. From Eqs. (13) and (16), it follows that $w_{LBw} = w_{LBw} = (1 - q)r$. Therefore is $Z_{LBw} - Z_{LBw} = 0$, the sizes of the inputs are equal in Regime $LwB$ and $LBw$, and $X_{LwB} = X_{LBw}$. However, since $w_{LBw} > w_{LBw}$ when $\gamma_n > 0$, it follows that $B_{LBw} > B_{LBw}$ and $C_{LBw} < C_{LBw}$. Thus, $V_{LwB} > V_{LBw}$. Regarding the union, $U_{LwB} > U_{LBw}$ because $L_{LwB} = L_{LBw}$ and $w_{LwB} > w_{LBw}$.

To compare the utility levels of the sponsor in Regime $BLW$ and C, I need the values of $L$, $Z$ and $C$ in the two regimes. $L_C$ is given by Eq. (22). Utilization of Eqs. (5), (9), (21), (22) and (A2) gives
\[
Z_C = (1 - \alpha) \beta \frac{1 - \alpha \beta + \gamma_L \sigma}{1 + \gamma_L} \frac{1 - \alpha \beta + \gamma_n \sigma}{1 + \gamma_n} Y.
\]
\[(A16)\]
\[
C_C = \frac{1 - \beta}{1 + \gamma_L} \frac{1 - \alpha \beta + \gamma_L \sigma}{1 - \alpha \beta + \gamma_n \sigma}.
\]
\[(A17)\]
For Regime $BLW$, I need the value of $B$. The sponsor’s decision problem is to maximize utility (4) with respect to $B$ and $C$, subjected to Eqs. (1) and (5), treating the wage and employment as endogenous. The first order condition can be written
\[
(1 - \beta) Z_{BLw} = (1 - \alpha) \beta C_{BLw} + C_{BLw} \left[ \frac{1}{BLw} \frac{dL_{BLw}}{dB_{BLw}} \left[ \alpha \beta Z_{BLw} \right] - (1 - \alpha) \beta L_{BLw} \frac{dL_{BLw}}{dB_{BLw}} \right].
\]
\[(A18)\]
Combining the employment reaction–function (A11) and the wage reaction–function (A7) yields the wage as Eq. (13). Consequently, \( \frac{dw_{BLw}}{dB_{BLw}} = 0 \). From Eq. (A11) it follows that \( \frac{1}{L_{BLw}} \cdot \frac{dL_{BLw}}{dB_{BLw}} = \frac{1}{B_{BLw}} \). These results plugged into Eq. (A18) together with the budget constraints (2) and (5) give after some manipulation

\[
B_{BLw} = \beta Y. \tag{A19}
\]

Utilization of Eqs. (2), (5), (13), (A11) and (A19) gives

\[
L_{BLw} = \frac{\alpha \beta + \gamma_L (1 - \sigma)}{\beta + \gamma_L} \cdot Y, \tag{A20}
\]

\[
Z_{BLw} = \beta \frac{(1 - \alpha) \beta (1 - \sigma)}{\beta + \gamma_L} \frac{\gamma_L \sigma}{(1 - \alpha) \beta + \gamma_L \sigma} \cdot Y, \tag{A21}
\]

\[
C_{BLw} = (1 - \beta) Y. \tag{A22}
\]

Plugging the values of \( L, Z \) and \( C \) into the utility function of the sponsor gives after some manipulation that \( V_{BLw} > V_C \) if \( K^{(i)} > 1 \) where

\[
K^{(ii)} = (1 + \gamma_L) \left( \frac{\beta}{\beta + \gamma_L} \right)^{\beta} \left( \frac{1 - \alpha \beta + \gamma_L \sigma}{1 - \alpha \beta + \gamma_L \sigma} \right)^{(1 - \alpha) \beta} \times \left( \frac{(1 - \alpha) \beta + \gamma_L \sigma}{(1 - \alpha) \beta + \gamma_L \sigma} \right)^{(1 - \alpha) \beta}. \tag{A23}
\]

Inspection of Eq. (A23) yields that \( K^{(ii)} = 1 \) if \( \gamma_n = \gamma_L = \gamma = 0 \). Because \( K^{(ii)} \) is increasing in \( \gamma \), \( V_{BLw} > V_C \) if \( \gamma_n = \gamma_L > 0 \). \( K^{(ii)} \) is also increasing in \( \gamma_n \). This proves that \( V_{BLw} > V_C \) if \( \gamma_n \geq \gamma_L \) when \( \gamma_n > 0 \).

Regarding the union, Proposition 6 says that \( w_{BLw} < w_C \) if \( \gamma_n > \gamma_L \) and \( w_{BLw} = w_C \) if \( \gamma_n = \gamma_L \). It follows from Eqs. (22) and (A20) that \( L_{BLw} < L_C \) if \( \gamma_L > 0 \) and \( L_{BLw} = L_C \) if \( \gamma_L = 0 \). This proves that \( U_{BLw} < U_C \) if \( \gamma_n \geq \gamma_L \) when \( \gamma_n > 0 \), and ends the proof of part (ii).

To calculate \( \hat{\sigma} \), I need the value of \( Z \) and \( C \) in Regime LBw. Utilization of Eqs. (5), (14), (15), (16) and (A9) gives

\[
Z_{LBw} = q \frac{(1 - \alpha) \beta (1 - \alpha) \beta + \gamma_L \sigma}{\beta + \gamma_L} \cdot Y, \tag{A24}
\]

\[
C_{LBw} = (1 - q) \frac{(1 - \alpha) \beta + \gamma_L \sigma}{\beta + \gamma_L} \cdot Y. \tag{A25}
\]

Plugging the values of \( L, Z \) and \( C \) into the utility function of the sponsor gives after some manipulation that \( V_{LBw} > V_{BLw} \) if

\[
\left( \frac{1}{\beta} \right)^{\beta} q^{(1 - \alpha) \beta} \left( \frac{1 - q (1 - \alpha) \beta + \gamma_L \sigma}{1 - \beta} \right)^{(1 - \beta)} > 1. \tag{A26}
\]
Collecting terms gives
\[ \gamma_L \sigma > K^{(iii)}(\beta + \gamma_L) - (1 - \alpha)\beta, \]
where
\[ K^{(iii)} = (\beta) \frac{1 - \alpha}{1 - \alpha} (1 - \alpha \beta) \frac{1 - \alpha}{1 - \alpha} (1 - \alpha \beta) . \]  
\[ (A27) \]

Now \( \bar{\sigma} = 1 / K^{(iii)}(\beta + \gamma_L) - (1 - \alpha)\beta / \gamma_L, \) and \( V_{LBV} > V_{BLV} \) if \( \sigma > \bar{\sigma}. \) It is easy to see that \if \gamma_L \rightarrow \infty, \bar{\sigma} = K^{(iii)} > 0. \) Inspection of the expression of \( K^{(iii)} \) yields that \( K^{(iii)} = 1 \) if \( \alpha = 0, \) and that \( K^{(iii)} \) is decreasing in \( \alpha. \) Consequently, \( K^{(iii)} < 1 \) if \( \alpha > 0, \) and \( 0 < \bar{\sigma} < 1 \) if \( \gamma_L \rightarrow \infty. \) It is also possible to show that \( d\bar{\sigma}/d\gamma_L < 0. \) Thus, \( \bar{\sigma} \rightarrow \infty \) if \( \gamma_L \rightarrow 0. \) This ends the proof of part (iii).

Lastly, by plugging the values of \( w \) and \( L \) into the union utility function, \( U_{LB} > U_C \) if \( K^{(iv)} > 1 \) where
\[ K^{(iv)} = \frac{1 + \gamma_L}{\beta + \gamma_L} \left( \frac{(1 - \alpha)\beta + \gamma_L \sigma}{1 - \alpha \beta + \gamma_L \sigma} \right)^{1 + \gamma_L \sigma}. \]  
\[ (A28) \]

It is easy to see that a sufficient condition for \( U_{LB} > U_C \) is \( \gamma_L \leq \gamma_L. \) \( \bar{\sigma}^u \) is the value of \( \sigma \) that implies \( K^{(iv)} = 1, \) and \( U_{LB} > U_C \) if \( \sigma < \bar{\sigma}^u. \) Consider the case when \( \gamma_L = 0 \) and \( \gamma_u \rightarrow \infty. \) Then
\[ \bar{\sigma}^u = \frac{\ln \beta}{\ln(1 - \alpha) - \ln(1 - \alpha \beta)}. \]  
\[ (A29) \]
Consequently, there exist values of \( \gamma_u \) and \( \gamma_L, \) and \( \gamma_u > \gamma_L, \) for which \( \bar{\sigma}^u \) exists. In fact, \( \bar{\sigma}^u < 1, \) in this particular case. This ends the proof.

References


